Robust quantum repeater with atomic ensembles against phase and polarization instability

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(Dated: March 11, 2009)

We propose an alternative scheme for quantum repeater without phase stabilization and polarization calibration of photons transmitted over long-distance channel. We introduce time-bin photonic states and use a new two-photon interference configuration to robustly generate entanglement between distant atomic-ensemble-based memory qubits. Our scheme can be performed with current experimental setups through making some simple adjustments.

PACS numbers: 03.67.-a, 03.67.Mn, 03.67.Pp

I. INTRODUCTION

The concept of quantum repeater [1]was introduced for long-distance quantum communication in order to overcome the problems caused by inevitable photon loss in the transmission channel. Generating distant entanglement is a crucial ingredient of a quantum repeater protocol. In 2001, Duan, Lukin, Cirac and Zoller (DLCZ) proposed a original scheme [2] to use atomic-ensembles and linear optics in which robustly generating entanglement over long distances can be achieved. Motivated by the DLCZ protocol, much experimental effort[3, 4, 5] has been made in the last few years.

However, it has been shown that DLCZ protocol requires severe phase stability since entanglement generation and entanglement swapping in the protocol depend on single-photon Mach-Zehnder type interferences[6], and this problem make its experimental realization extremely difficult by the current technology. Hence a novel scheme[6] based on DLCZ scheme was proposed to use phase-insensitive two-photon quantum interference which dramatically relax this phase stability requirements. A latest experiment[7] has primarily demonstrated the scheme over a distance of 300 meters. Meanwhile, a fast and robust approach [8] was proposed and also can solve the phase stability problem as long as the entanglement generation is performed locally, and they gave a detailed comparisons between the schemes. In addition, there are another novel schemes [9, 10] proposed to improve the efficiency of quantum repeaters.

These protocols greatly stimulate experimental implementations of quantum repeater, but there are still some problems need to be solved, such as reliable transmission of photon's polarization states over noisy channel. Since the photon interferences rely on the polarization states, the ability to maintain photonic polarizations is indispensable in the process of distant entanglement generation or swapping. Most of the time, optical fibers are used as photon transmission channel. Due to the fiber birefringence, the photonic polarizations will be changed randomly [11]. Experimentally, active feedback compensation could be applied to solve this problem[12], but it is efficient only when the thermal and mechanical fluctuations are rather slow. Furthermore, even though polarization compensations can be used efficiently, imperfect shared reference frame (SRF) for polarization orientation may cause some errors. It is difficult to correct this kind of errors since establishing a perfect SRF requires infinite communication[13]. Due to these reasons, it would be better to have a quantum repeater scheme with inherent polarization insensitivity.

In this paper, we propose an alternative approach to create distant entanglement between atomic-ensembles. In our scheme neither the phase stabilizing nor the polarization calibrating is needed for photons transmitted over long distance. Through introducing time-bin photonic states and using a new two-photon interference configuration, we make only the unchanged part of initial polarization states contribute to the desired results. Combined with local entanglement swapping for entanglement connection, our scheme can be used to implement a robust quantum repeater.

II. ENTANGLEMENT GENERATION

Here optical thick atomic ensemble, which includes N atoms with Λ level structure (see inset of Fig.1), is used as quantum memory. Each atomic ensemble is illuminated by a short, off-resonant write pulse that induces a spontaneous Raman process. This process will produce a forward-scattered Stokes light and a collective atomic excitation state[2]. The photon-atom system can be described as(neglecting the higher-order terms)

$$[1+\sqrt{\chi}S^{\dagger}a^{\dagger}+\frac{\chi}{2}(S^{\dagger}a^{\dagger})^2]|vac\rangle, \tag{1}$$

where $|vac\rangle$ denotes that all the ensemble atoms are in the ground state $|g\rangle$ and the Stokes light in the vacuum state, $a^{\dagger}$ is the creation operator of the Stokes light, and the collective atomic excitation is defined by

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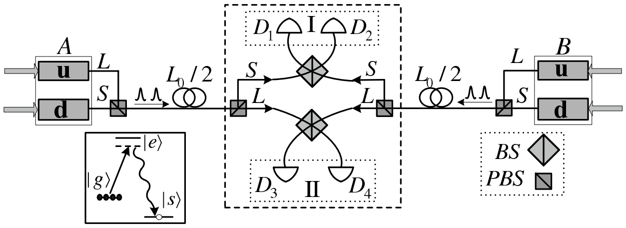

FIG. 1: Schematic of our scheme. At each node, the Stokes photons generated from the two atomic ensembles have orthogonal polarizations, i.e.$|H\rangle$ and $|V\rangle$. The PBS transmits $|H\rangle$ and reflects $|V\rangle$. Before leaving the node, the photon wave packets with different polarizations experience different paths and then travel to the middle point where they experience a same path difference but with opposite arrangement. There are two detection zones: one with the single photon detectors $D_{1,2}$ and the other with $D_{3,4}$. A coincidence count between the detectors at either detection zone, e.g. D_1 and D_3, will project the four atomic ensembles into a complex entangled state up to a local unitary transformation. The inset shows the ensemble atom energy level.

$S^\dagger = \frac{1}{\sqrt{N}} \sum_{i=1}^{N} |s\rangle_i \langle g|$. The excitation probability $\chi \ll 1$ can be achieved by manipulating the write laser pulse.

At each communication node, two atomic ensembles are used to encode a memory qubit(see the Fig.1). The two ensembles are excited simultaneously by write laser pulses, and the Stokes photons generated from them have orthogonal polarization states, i.e. $|H\rangle$ and $|V\rangle$, which denote horizontal and vertical linear polarization, respectively. The state $|H\rangle$ propagates through a short path (S) and the state $|V\rangle$ goes through a long path (L), and are combined at a polarization beam splitter (PBS) which transmits $|H\rangle$ and reflects $|V\rangle$. Therefore photon wave packets with different polarizations correspond to different time bins. As long as the path difference is less than the photons coherence length (about $3m$ or more for photons generated from atomic ensembles[14]), after the PBS the memory qubit is effectively entangled with the polarization and the time-bin states of the emitted Stokes photons. The state of the atom-photon system can be written as

$$
\begin{aligned}
|\psi\rangle = \{1 + \sqrt{\chi}(S_u^\dagger a_{V,L}^\dagger + S_d^\dagger a_{H,S}^\dagger) + \frac{\chi}{2}[(S_u^\dagger a_{V,L}^\dagger)^2 \\
+ 2S_u^\dagger a_{V,L}^\dagger S_d^\dagger a_{H,S}^\dagger + (S_d^\dagger a_{H,S}^\dagger)^2]\}|vac\rangle,
\end{aligned}
\tag{2}
$$

where the subscripts u and d are used to distinguish the two ensembles, and $a_{V,L}^\dagger (a_{H,S}^\dagger)$ denotes the creation operator of the Stokes photon with vertical (horizontal) polarization passing through the long (short) path.

Assume two neighboring communication nodes, denoted by A and B, are connected through certain transmission channel such as optical fiber with a distance of L_0. At the middle point between the two nodes, the Stokes photons are firstly directed to PBS. Due to channel noise or imperfect SRF, the polarization states of photons arrived at the middle point have been changed, and as long as the time intervals between the subsequent transmitted photon wave packets are small, it is reasonable to assume that the polarization changes are the same to them. After the PBS, there are also a long path and a short one whose difference is the same as the one at each node. The polarization transformation during this process are usually considered as a unitary transformation[11], so the evolution of the photonic polarization components can be described as:

$$
\begin{aligned}
a_{H,S}^\dagger & \xrightarrow{\ noise\ } \cos\theta a_{H,S}^\dagger + e^{i\varphi}\sin\theta a_{V,S}^\dagger \\
& \xrightarrow{\ PBS\ } \cos\theta a_{H,SL}^\dagger + e^{i\varphi}\sin\theta a_{V,SS}^\dagger,
\end{aligned}
\tag{3}
$$

$$
\begin{aligned}
a_{V,L}^\dagger & \xrightarrow{\ noise\ } - e^{-i\varphi}\sin\theta a_{H,L}^\dagger + \cos\theta a_{V,L}^\dagger \\
& \xrightarrow{\ PBS\ } - e^{-i\varphi}\sin\theta a_{H,LL}^\dagger + \cos\theta a_{V,LS}^\dagger,
\end{aligned}
\tag{4}
$$

where θ and φ are random noise parameters. Obviously, there are four time bins SS, SL, LS and LL for each Stokes photon, and SL, LS are the same time bin. Now the atom-photon system can be described as:

$$
|\psi\rangle = |vac\rangle + |\psi_{LS,SL}\rangle + |\psi_{SS,LL}\rangle + |\psi_{cross}\rangle, \tag{5}
$$

with

$$
\begin{aligned}
|\psi_{LS,SL}\rangle = & \{\cos\theta\sqrt{\chi}(S_u^\dagger a_{V,LS}^\dagger + S_d^\dagger a_{H,SL}^\dagger) \\
& + \frac{\chi}{2}\cos^2\theta[(S_u^\dagger a_{V,LS}^\dagger)^2 + (S_d^\dagger a_{H,SL}^\dagger)^2 \\
& + 2S_u^\dagger S_d^\dagger a_{V,LS}^\dagger a_{H,SL}^\dagger]\}|vac\rangle,
\end{aligned}
\tag{6}
$$

$$
\begin{aligned}
|\psi_{SS,LL}\rangle = & \{\sqrt{\chi}\sin\theta(S_d^\dagger a_{SS}^\dagger - S_u^\dagger a_{LL}^\dagger) \\
& + \frac{\chi}{2}\sin^2\theta[(S_u^\dagger a_{LL}^\dagger)^2 + (S_d^\dagger a_{SS}^\dagger)^2 \\
& - 2S_u^\dagger S_d^\dagger a_{LL}^\dagger a_{SS}^\dagger]\}|vac\rangle,
\end{aligned}
\tag{7}
$$

$$
\begin{aligned}
|\psi_{cross}\rangle = & \chi\sin\theta\cos\theta[(S_d^\dagger)^2 a_{SL}^\dagger a_{SS}^\dagger - (S_u^\dagger)^2 a_{LL}^\dagger a_{LS}^\dagger \\
& + S_u^\dagger S_d^\dagger (a_{LS}^\dagger a_{SS}^\dagger - a_{LL}^\dagger a_{SL}^\dagger)]\}|vac\rangle,
\end{aligned}
\tag{8}
$$

Here for simplicity the φ and the polarization subscripts are not visibly expressed in the states $|\psi_{SS,LL}\rangle$ and $|\psi_{cross}\rangle$ without any influences on the subsequent analysis.

The setup for photon interferences at the middle point (see Fig.1) is that the photons after the PBS are directed into beam splitters (BS) followed by single photon detectors which are turned on only at the time $SL(LS)$. Therefore the terms only including time bins SS and LL will not have any contributions to the detection results and can be safely neglected. To generate entanglement between the nodes A and B, laser pulses excite the ensembles in both nodes simultaneously, and the whole system is described by the state $|\psi\rangle_A \bigotimes |\psi\rangle_B$, where $|\psi\rangle_A$ and $|\psi\rangle_B$ are given by equation (5) with all the operators

and states distinguished by the subscript A and B. A coincidence count between the detectors at either detection zone, e.g. D_1 and D_3, will project the neighboring memory qubits into a complex state with contributions from second-order excitations. Note that in second order in χ, the states $|\psi_{cross}\rangle_{A,B}$ can just trigger one detector at either detection zone at the time $SL(LS)$ and consequently have no contribution to the coincidence. Thus a coincidence count between detectors, for instance D_1 and D_3, projects the two memory quits into

$$|\psi\rangle_{AB} = \frac{1}{2}[e^{i(\phi+\phi')}\cos\theta\cos\theta'(S^\dagger_{u_A}S^\dagger_{d_B} + S^\dagger_{u_A}S^\dagger_{d_B}) \\ e^{2i\phi}\cos^2\theta S^\dagger_{u_A}S^\dagger_{d_A} + e^{2i\phi'}\cos^2\theta' S^\dagger_{u_B}S^\dagger_{d_B}]|vac\rangle \quad (9)$$

where $\theta(\theta')$ and $\phi(\phi')$ are the noise parameter and the polarization-independent phase that the photons acquires during the transmission from the node $A(B)$ to the middle point, respectively. The first part of the state is the desired maximally entangled state. The second part is unwanted two-excitation state and can be effectively eliminated by entanglement swapping. The success probability is on the order of $O(\chi^2\eta^2 e^{-L_0/L_{att}})$, where η is the detection efficiency and L_{att} is the channel attenuation length. Until now, we have generated the state similar to the one (Eq.(S3) in Supplementary information of Ref.[7]) which was created in the recent experimental demonstration of quantum repeater over a distance of 300 meters. It is obvious that the polarization noises only influence the success probability but have no effect on the fidelity of the desired state, and the phases ϕ and ϕ' only lead to a trivial global factor $e^{i(\phi+\phi')}$ on the desired state.

III. LOCAL ENTANGLEMENT SWAPPING FOR ENTANGLEMENT CONNECTION

The entanglement swapping setup is depicted in Fig.2, which is the same as the one in the schemes[6]. Note that there is no path difference at this situation. Consider three communication nodes A, B and C, and assume that we have created the complex entangled states (given by Eq.(9)) $|\psi\rangle_{AB_L}$ and $|\psi\rangle_{B_RC}$ both in (A, B_L) and in (B_R, C), respectively. The memory qubits B_L and B_R at node B are illuminated simultaneously by retrieval laser pulses. The retrieved anti-Stokes photons are subject to Bell-state measurement (BSM) which is used to eliminate the two-excitation terms since the arrangement of the PBSs is to identify $|\phi^\pm\rangle$at $|+\rangle/|-\rangle$ polarization basis, where $|\phi^\pm\rangle = \frac{1}{\sqrt{2}}(|+\rangle|+\rangle \pm |-\rangle|-\rangle)$. Therefore the two-photon states generated from the unwanted two-excitation terms are directed into the same detection zone and will not induce any coincidences. In addition, the capability of distinguishing photon numbers is technically demanding, and the retrieve efficiency is determined by the optical depth of the atomic ensembles[15]. Taking into account of these imperfections, the coincidence counts in BSM actually prepare the memory

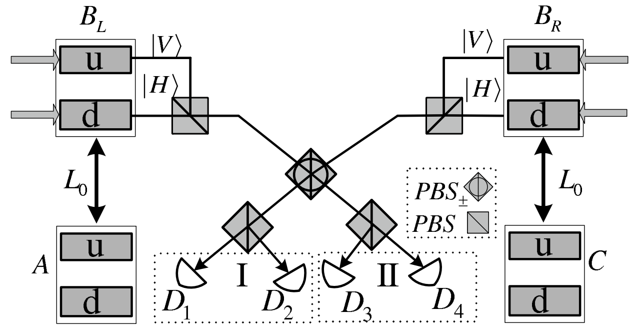

FIG. 2: Set-up for local entanglement swapping. There are three nodes A, B and C. At node B, the anti-Stokes photons retrieved from memory qubits B_L and B_R have orthogonal polarizations $|H\rangle$ and $|V\rangle$. The $PBS(PBS_\pm)$ transmits $|H\rangle(|+\rangle)$ and reflects $|V\rangle(|-\rangle)$, where$|\pm\rangle = \frac{1}{\sqrt{2}}|H\rangle \pm |V\rangle$. The $D_{1,2,3,4}$ are single photon detectors at two detection zones.

qubits into a mixed entangled state of the form $\rho_{AC} = p_2\rho_2 + p_1\rho_1 + p_0\rho_0$, where the coefficients p_2, p_1 and p_0 are determined by the retrieval efficiency and detection efficiency[6]. Here $\rho_2 = |\phi\rangle_{AC}\langle\phi|$ is the maximal entangled state, where $|\phi\rangle_{AC} = \frac{1}{\sqrt{2}}(S^\dagger_{u_A}S^\dagger_{u_C} + S^\dagger_{d_A}S^\dagger_{d_C})|vac\rangle$, ρ_0 is the vacuum state that all the atomic ensembles are in the ground states, and ρ_1 is a maximally mixed state in which only one of the four atomic ensembles has one excitation.

Although ρ_{AC} still includes unwanted terms, it can be projected automatically to maximally entangled states in the entanglement-based quantum cryptography schemes. When implementing quantum cryptograph via Ekert protocol[16], only the coincidence counts between the detectors at two remote nodes are registered and used for quantum cryptography. Therefore only the maximally entangled state in ρ_{AC} will contribute to the experimental results. In this sense, ρ_{AC} is equivalent to the Bell state $|\phi\rangle_{AC}$.

To implement a quantum repeater protocol, further entanglement swapping is required. Since the noise parameters only change the coefficients of the obtained states, the analysis of the entanglement connection in Ref.[6] can be directly used here, that is, as long as the excitation probability χ is small enough, the contributions from the higher-order excitations can be safely neglected. The probability to find an desired entangled pair in the remaining memory qubits is almost a constant and will not decrease significantly during the entanglement connection process.

IV. CONCLUSION

In the existing quantum repeater protocols, the ability to reliably transfer of photon's polarization is indispensable, but it is not easy to meet the prerequisite in practice. For this reason, we have proposed an alternative approach with inherent polarization insensitivity to

generate entanglement between distant communication nodes. In our scheme, neither the phase stabilizing nor the polarization calibrating is needed for photons transmitted over long distance. Through introducing path difference, we make photon wave packets with different polarizations correspond to different time-bins, and use the two-photon interference with different configuration to generate entanglement between remote communication nodes. Hence only the unchanged part of initial polarization states can induce the coincidences between detectors, so that the polarization noise only have influence on the success probability but the fidelity of the desired states. Consider the function of atom ensembles as quantum memory for entanglement of a storage time up to milliseconds[17], the capacity of creating high-fidelity entangled states may be preferred even if at the cost of some efficiencies. Combined with local entanglement swapping for entanglement connection, our scheme can be used to implement a robust quantum repeater. Comparing with the DLCZ scheme and the scheme of Ref.[6], the generated entangled states in our scheme are of higher fidelity.

Finally, it is pointed out that our scheme can be used to perform remote entanglement swapping after locally generating entanglement, which would make the quantum repeater scheme more efficient[10]. For instance, the local entangled state (eq.(10) in Ref.[6]) generated via the setup in Fig.2 may be robustly connected to longer communication distance by use of our scheme.

Acknowledgments

we thank Cheng-Xi Yang, Lin Yang and Xian-Min Jin for useful discussions. X.B.Wang was supported in part by the National Basic Research Program of China grant No. 2007CB907900, and 2007CB807901, NSFC grant No. 60725416 and China Hi-Tech program grant No. 2006AA01Z420. L.-M. Liang was supported by National Funds of Natural Science grant No. 10504042.

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